TWO-UNIT STANDBY SYSTEM WITH IMPERFECT SWITCHING AND DISCRETE FAILURE AND REPAIR TIMES

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ABSTRACT: Generally consider that failure and repair times of a unit are continuous distribution. But, in practice the situation exist when the failure and repair of a unit occur at discrete random epoch so that the life time and repair time of a unit follow discrete distribution, like geometric, negative binomial, Poisson etc. actually, discrete failure data arise in several common situations for example in a photo copy machine the bulb is lightened every time when a copy is taken. Thus, the life time of the bulb is discrete random variable.

Keeping the above concept of discrete time modelling we in the present paper analyse a two, identical unit is via partial system with imperfect switching. Initially one unit is operative and othet is stand by, we consider two types of failures, partial failure and total failure. Failure and time of the unit are considered as Geometric distribution. Various measures of system effectiveness are also obtained.

Keywords: Geometric distribution, redundant system, discrete random variable.

1. INTRODUCTION

In some situation, discrete failure time distributions are appropriate to model "lifetimes". For example, discrete distribution is appropriate when a piece of equipment operates in cycles and the number of cycles prior to failure is observed.

Discrete failure data arise in several common situations, for example: a) A device is monitored only once per time period (e.g, an hour, a day), and the observation is the number of time periods successfully completed prior to failure of the device. b) A piece of equipment operates in cycles and the experimenter observes the number of cycles successfully completed prior to failure.

In situations where the observed data values are very large (in thousands of cycles, etc.) a continuous distribution is an adequate model for the discrete random variable. However when the observed values are small, continuous distribution might not adequately describe a discrete random variable.

The purpose of this paper is to incorporate the concept of discrete distribution. Generally consider that failure and repair times of a unit continuous distribution of time to failure and time to repair of a unit. However, in practice the situation exists when the failure and repair of a unit occur at discrete random epochs so that the life time and repair time of a unit follows discrete distribution like geometric, negative binomial, poission etc.

Some examples of discrete life times are as follows:

"In a photo copy machine the bulb is lightened every time when a copy is taken. Thus, the life time of the bulb is a discrete random variable".

"In an on /off switching device, the life time of the switch is a discrete random variable".

"In a refrigerator, the bulb is lightened whenever the door of the refrigerator is opened. Thus, the life time of the bulb is the discrete random variable".

Thus, we see that many practical situations of importance are represented with the help of discrete life time models. Similarly, the repair time may be considered as discrete random variable as dividing the whole time interval into various small parts of time. Discrete time models are considered by several researchers including Padgett and Spurrier [1985], Salvia and Bollinger [1982], J.F.Lawless [1982], Kalbfleish and Prentice [1980] etc.

This model consists of two-unit, the main unit and the standby unit. It is assume that main unit can work in normal mode i.e. with full efficiency. The single repairman facility is available with the system to repair the failed unit and repair the switch. First priority of the repairman is repair to the failed switch. For both the units, the distribution of time to failure and repair is taken as geometric distribution and repair time distribution of the switch is also geometric distribution.

Using regenerative point technique, the following reliability characteristics of interest have been obtained.

(i)   Distribution of time to system failure and its mean.

(ii)   Pointwise and steady state availability of the system

(iii)   Expected busy period of repairman in [0, t] and in steady state.

(iv)   Expected profit incurred in [0, t] and in steady state.

Few characteristics such as MTSF, system availability and profit in steady state have also been studied through graphs.

2. System description and Assumption for Model

System is analysed under the following assumptions.

(i)   A cold standby system consists of two-identical unit one is operative and other is standby. Each unit has two-modes: normal (N), and total failure (F). The standby-unit cannot fail.

(ii)   Failure is self announcing state operating.

(iii)   Switching is imperfect in the transition from standby state to operating state.

(iv)   A single repairman is available to repair a failed unit and switch.

(v)   The failure and repair time distribution of the unit are independent having geometric distribution with parameter p and r_2.

(vi)   The repair time distribution of the switch is independent having geometric distribution with parameter r_1.

(vii)   The failure times of the unit are taken as independent random variable.

2.1 Notations and States of the Systems

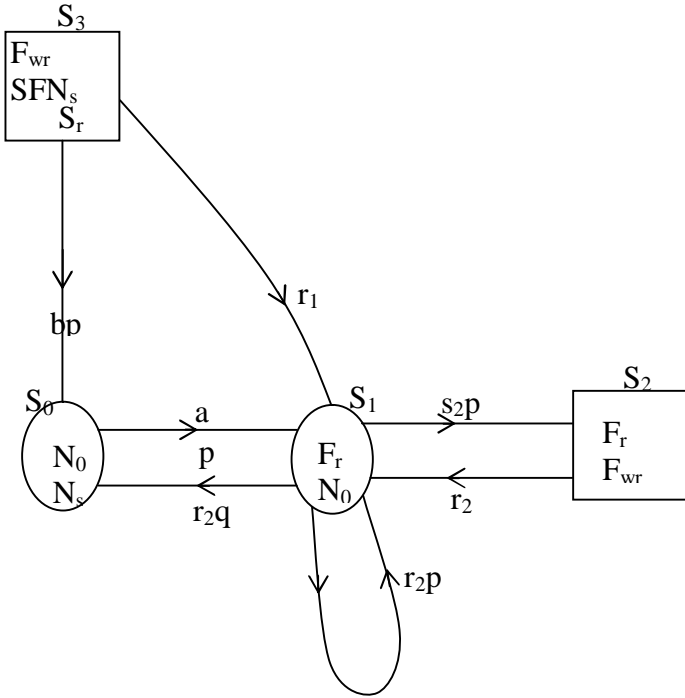|		
N_0	:	unit in normal mode and operative
N_s	:	unit in normal mode and standby
$F_r \mid F_{wr}$	:	unit in failure mode and under repair | waiting for repair
S_r	:	Switch failed and under repair
SFN_s	:	Switch failed unit in standby state

Up State   **Down State**

$S_0 \equiv (N_0, N_s)$, $S_1 \equiv (F_r, N_0)$ $S_2 \equiv (F_r, F_{wr})$, $S_3 \equiv (F_{wr}, SFN_s)$

Other symbols

a : The probability that switch is perfect

b : The probability that switch is imperfect

Operative state

Failed state

Transition Diagram

2.2 Transition probabilities and Sojourn Times

$$Q_{01}(t) = ap\frac{1 - q^{(t+1)}}{1 - q} \qquad Q_{03}(t) = bp\frac{1 - q^{(t+1)}}{1 - q}$$

$$Q_{01}(t) = {}^{r}_2q\frac{[1 - (qs_2)^{t+1}]}{1 - qs_2} \qquad Q_{11}(t) = {}^{r}_2p\frac{[1 - (qs_2)^{t+1}]}{1 - qs_2}$$

$$Q_{12}(t) = {}^{ps_2}\frac{[1 - (qs_2)^{t+1}]}{1 - qs_2} \qquad Q_{21}(t) = {}^{r}_2\frac{[1 - s_2^{(t+1)}]}{1 - s_2}$$

$$Q_{31}(t) = {}^{r}_1\frac{[1 - s_1^{(t+1)}]}{1 - s_1} \qquad\qquad (1.1 - 1.7)$$

The steady state transition probabilities from state S_i to S_j can be obtained from

$$P_{ij} = \lim_{t \to \infty} Q_{ij}$$

$$P_{01} + P_{03} = 1 \qquad P_{10} + P_{11} + P_{12} = 1 \qquad P_{21} = P_{31} = 1$$

2.3 Mean Sojourn Times

Let T_i be the sojourn time in state $S_i (i = 0, 1)$ then mean sojourn time in state S_i is given by

$$\mu_i = \sum_{t=0}^{\infty} P(T_i > t)$$

so that

$$\mu_0 = \frac{1}{1 - q} \qquad\qquad \mu_1 = \frac{1}{1 - s_2 q}$$

$$\mu_2 = \frac{1}{1 - s_2} \qquad\qquad \mu_3 = \frac{1}{1 - s_1} \qquad (1.8 \text{ -}1.11)$$

Defining m_{ij} as the mean sojourn time of the system in state S_i when the system is transit into state S_j i.e.

$$m_{ij} = \sum_{t=0}^{\infty} t\, q_{ij}(t)$$

$$m_{01} + m_{02} = \mu_0 \qquad\qquad m_{10} + m_{11} + m_{12} = qs_2\,\mu_1$$

$$m_{31} = s_1\,\mu_3 \qquad\qquad m_{21} = s_2\,\mu_2$$

3 Reliability and Mean Time to System Failure

To obtain the distribution of the time to system failure we regard the failed states as absorbing states. By using probabilistic argument, the following recursive relation for $R_i(t)$:

$$R_0(t) = z_0(t) + q_{01}(t-1)\copyright R_1(t-1)$$

$$R_1(t) = z_1(t) + q_{10}(t-1)\copyright R_0(t-1) + q_{11}(t-1)\copyright R_1(t-1) \qquad (1.12 - 1.13)$$

Taking geometric transform on both sides

We get

$$R_0^*(h) = \frac{N_1(h)}{D_1(h)}$$

where,

$$N_1(h) = \begin{vmatrix} Z_0^*(h) & -hq_{01}^*(h) \\ Z_1^*(h) & 1-hq_{11}^*(h) \end{vmatrix}$$

$$N_1(h) = Z_0^*(h)[1-hq_{11}^*(h)] + hZ_1^*(h)q_{01}^*(h)$$

and

$$D_1(h) = \begin{vmatrix} 1 & -hq_{01}^*(h) \\ -hq_{10}^*(h) & 1-hq_{11}^*(h) \end{vmatrix}$$

$$D_1(h) = 1-hq_{11}^*(h) - h^2 q_{01}^*(h)q_{10}^*(h)$$

Then $\quad \text{MTSF} = \lim_{h \to 1} \dfrac{N_1(h)}{D_1(h)} - 1$

$$\text{MTSF} = \frac{N_1}{D_1} \qquad\qquad (1.14)$$

where

$$N_1 = \mu_0(1 - P_{11}) + \mu_1 P_{01} + P_{11} + P_{01}\, P_{10} - 1$$

$$D_1 = 1 - P_{11} - P_{01}\, P_{10}$$

4 Availability Analysis

Let $A_i(t)$ be the probability that the system is up at epoch t when it is initially started from regenerative state S_i by simple probabilistic arguments the following recurrence relations are obtained.

$A_0(t) = Z_0(t) + q_{01}(t-1)ⓒ A_1(t-1) + q_{03}(t-1)ⓒ A_3(t-1)$

$A_1(t) = Z_1(t) + q_{10}(t-1)ⓒ A_0(t-1) + q_{11}(t-1) ⓒ A_1(t-1) + q_{12}(t-1)ⓒ$
$\qquad A_2(t-1)$

$A_2(t) = q_{21}(t-1) ⓒ A_1(t-1)$

$A_3(t) = q_{31}(t-1) ⓒ A_1(t-1) \qquad\qquad (1.15 - 1.18)$

By taking geometric transformation on both sides and solving the equations.

$$A_0^*(h) = \frac{N_2(h)}{D_2(h)}$$

where

$N_2(h) \qquad\qquad\qquad\qquad\qquad\qquad =$

$$\begin{vmatrix} Z_0^*(h) & -hq_{01}^*(h) & 0 & -hq_{03}^*(h) \\ Z_1^*(h) & 1-hq_{11}^*(h) & -hq_{12}^*(h) & 0 \\ 0 & -hq_{21}^*(h) & 1 & 0 \\ 0 & -hq_{31}^*(h) & 0 & 1 \end{vmatrix}$$

$N_2(h) = Z_0^*(h)(1-hq_{11}^*(h)) - h^2 q_{12}^*(h)q_{21}^*(h)Z_0^*(h) + hq_{01}^*(h)Z_1^*(h)$
$\qquad + h^2\, q_{31}^*(h)q_{03}^*(h)Z_1^*(h)$

$$D_2(h) = \begin{vmatrix} 1 & -hq_{01}^*(h) & 0 & -hq_{03}^*(h) \\ -hq_{10}^*(h) & 1-hq_{11}^*(h) & -hq_{12}^*(h) & 0 \\ 0 & -hq_{21}^*(h) & 1 & 0 \\ 0 & -hq_{31}^*(h) & 0 & 1 \end{vmatrix}$$

$D_2(h) = 1-hq_{11}^*(h) - h^2 q_{12}^*(h)q_{21}^*(h) - h^2 q_{01}^*(h)q_{10}^*(h) - h^3 q_{03}^*(h)q_{10}^*(h)q_{31}^*(h)$

where,

$$Z_0(t) = q^t \qquad\qquad Z_1(t) = (sq)^t$$

The steady state availability of the system is given by

$$A_0 = \lim_{t \to \infty} A_0(t)$$

hence, by applying 'L' Hospital Rule we get

$$A_0 = \frac{N_2(1)}{D_2'(1)} \qquad\qquad (1.19)$$

$N_2(1) = \mu_0(1-P_{11} - P_{12}) + \mu_1(P_{01} + P_{03})$

$D_2'(1) = -\{qs_2\, \mu_1 + \mu_0\, P_{10} + P_{03}\, P_{10}\, s_1\mu_3 + P_{12}\, s_2\, \mu_2\}$

5 Busy Period Analysis

By probabilistic argument, we have the following recursive relation for B_i

$B_0(t) = q_{01}(t-1)ⓒ B_1(t-1) + q_{03}(t-1)ⓒ B_3(t-1)$

$B_1(t) = Z_1(t) + q_{10}(t-1)ⓒB_0(t-1) + q_{11}(t-1)ⓒ B_1(t-1) + q_{12}(t-1)ⓒB_2(t-1)$

$B_2(t) = Z_2(t) + q_{21}(t-1)ⓒ B_1(t-1)$

$B_3(t) = Z_3(t) + q_{31}(t-1)ⓒ B_1(t-1) \qquad (1.20-1.23)$

By taking geometric transform on both sides and solving the equations.

$$B_0^*(h) = \frac{N_3(h)}{D_2(h)}$$

where

$$N_3(h) = \begin{vmatrix} 0 & -hq_{01}^*(h) & 0 & -hq_{03}^*(h) \\ Z_1^*(h) & 1-hq_{11}^*(h) & -hq_{12}^*(h) & 0 \\ Z_2^*(h) & -hq_{21}^*(h) & 1 & 0 \\ Z_3^*(h) & -hq_{31}^*(h) & 0 & 1 \end{vmatrix}$$

$N_3(h) = h\,q_{01}^*(h)Z_1^*(h) + hq_{12}^*(h)Z_2^*(h) + h^2q_{03}^*(h)q_{31}^*(h)Z_1^*(h) + hq_{03}^*(h)$

$(1-hq_{11}^*(h))Z_3^*(h) + h^3q_{12}^*(h)q_{03}^*(h)q_{31}^*(h)Z_2^*(h) - h^3$

$q_{12}^*(h)\,q_{21}^*(h)\,q_{03}^*(h)Z_3^*(h)$

and $D_2(h)$ is the same as in availability analysis

$$B_0 = \lim_{t \to \infty} B_0(t)$$

Hence, by applying 'L' Hospital Rule, we get

$$B_0 = -\frac{N_3(1)}{D_2'(1)} \tag{1.24}$$

where

$N_3(1) = \mu_1 + \mu_2 P_{12}(1 + P_{03}) + \mu_3 P_{03}(1 - P_{11} - P_{12})$

and $D_2'(1)$ is the same as in availability analysis

6 Profit Function

The expected total profit in steady-state is

$$P = C_0 A_0 - C_1 B_0 \tag{1.25}$$

where

C_0 be the per unit up time revenue by the system and

C_1 be the per unit down time expenditure on the system.

7 Particular Case

$$P_{01} = \frac{ap}{1-q}, \qquad P_{03} = \frac{bp}{1-q}$$

$$P_{10} = \frac{r_2 q}{1-qs_2}, \qquad P_{11} = \frac{r_2 p}{1-qs_2},$$

$$P_{12} = \frac{ps_2}{1-qs_2} \qquad P_{21} = 1, \qquad P_{31} = 1$$

Using the above equation and equation (1.14), (1.19), (1.24) and (1.25) we can have the expression for MTSF, availability etc. for this particular case.

On the basis of the numerical values taken as:

P =0.01, r_1 = 0.7, r_2 = 0.25, a = 0.6, b = 0.4, s_1 = 0.3

The values of various measures of system effectiveness are obtained as:

Mean time to system failure (MTSF) =300

Availability (A_0) = 0.970397.

Busy period of analysis of repairman (B_0) = 0.040428

For the graphical interpretation, the mentioned particular case is considered.

Fig. 2 show the behaviour of MTSF with respect to failure rate (p). It is clear from the graph that the MTSF get decrease with increase in the value of failure rate.

Fig. 3 shows the behaviour MTSF with respect to repair rate (r_2). It is clear from the graph that the MTSF gets increase with increase in the value of repair rate.

Fig. 4 reveals the pattern of the profit with respect to failure rate (p) for different values of repair rate (r_2). The profit decreases with the increase in the value of failure rate (p) and is higher for higher values of repair rate (r_2). Following can also be observed from the graph:

(i) For r_2 = 0.25, P > or = or < 0 according as p < or = or > 0.0518. So, the system is profitable only if failure rate is less than 0.0518.

(ii) For r_2 = 0.5, P > or = or < 0 according as p < or = or > 0.0725. So, the system is profitable only if failure rate is less than 0.0725.

(iii)   For $r_2 = 0.75$, P > or = or < 0 according as p < or = or > 0.0878. Thus the system is not profitable when > 0.0878.

So, the companies using such systems can be suggested to purchase only those system which do not have failure rates greater than those discussed in points (i) to (iii) above in this particular case.
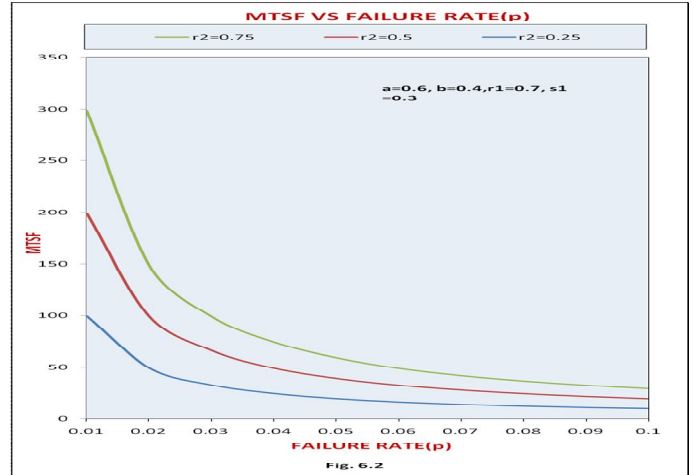
Fig. 1.5 shows the pattern of the profit with respect to repair rate (r_2) for different values of failure rate (p). The profit increases with the increase in the value of repair rate (r_2) and is lower for higher values of failure rate (p). Following can also be observed from the graph:
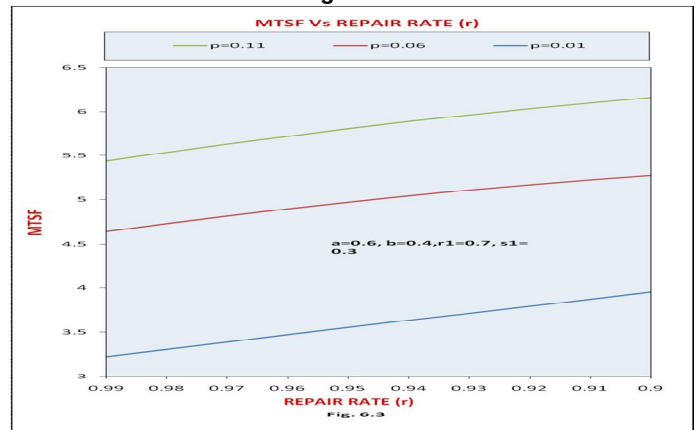
(i)   For p = 0.01, P > or = or < 0 according as r_2 > or = or < 0.06324. So, the system is profitable only if repair rate is greater than 0. 06234.

(ii)   For p = 0.21, P > or = or < 0 according as r_2 > or = or < 0.07374. Thus the system is not profitable when ≤ 0.07374.

So, the companies using such systems can be suggested to purchase only those systems which do not have repair rates greater than those discussed in points (i) to (ii) above in this particular case.
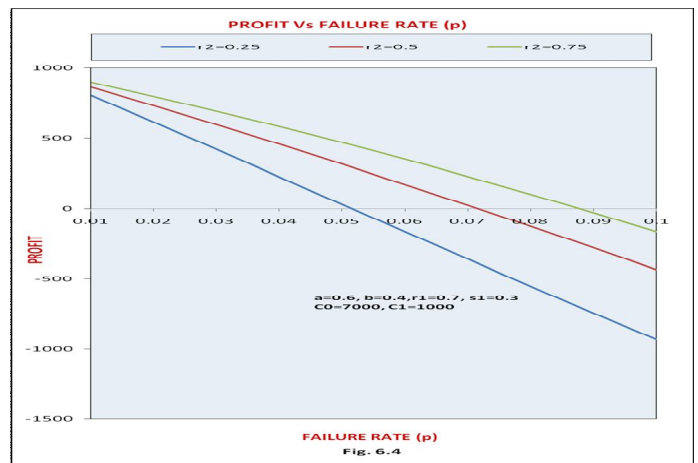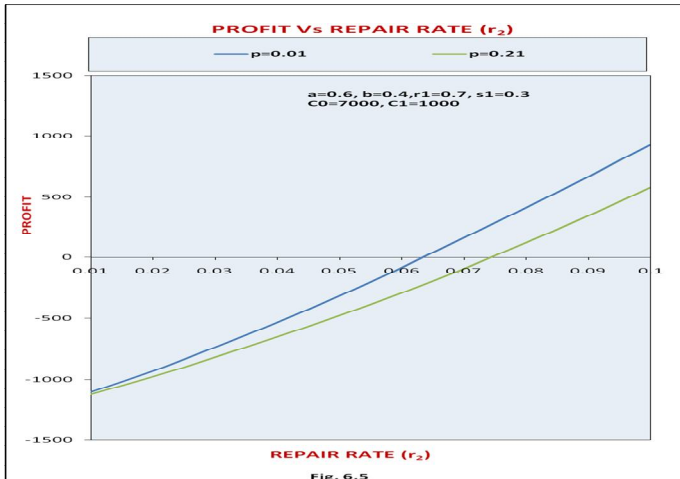

Figure: 2

Figure: 3

Figure: 4

Figure: 5

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